Boundary element analysis of the hydrodynamic forces in a 2D reservoir

J.L.Humar & A.M.Jablonski
Carleton University, Ottawa, Ontario, Canada

ABSTRACT: The boundary element method has been successfuly applied to the analysis of hydrodynamic forces in 2 and 3 dimensional finite reservoirs subjected to seismic motions. For infinite reservoirs, loss of energy due to pressure waves moving away towards infinity strongly influences the response. A boundary element solution has been attempted for this case by omitting the far boundary. Although this may give reasonable estimates of the hydrodynamic pressures, the results are sensitive to the length of the reservoir discretized and the convergence is not easily achieved. Another factor that may at times play an important role in controlling the hydrodynamic forces is foundation damping. This paper presents results of more recent work on the application of the boundary element method to the analysis of 2D reservoir vibration. Special boundary conditions to represent radiation damping at the far end as well as foundation damping have been incorporated in the formulation. Numerical results have been obtained and compared with classical results or results obtained by other researchers.

1 INTRODUCTION

Research on the seismic response of dam reservoir system has now established that both water compressiblity and dam flexibility significantly affect the hydrodynamic forces caused by earthquake motion. Other factors that influence the response are: radiation damping in an infinite reservoir due to energy transmitted by the outgoing waves, and the damping provided by flexible foundation material.

In analytical methods developed for the seismic analysis of gravity dams, the dam is usually represented as an assemblage of finite elements and the reservoir is modelled as a continuum with simple boundaries for which classical solutions can be easily obtained (Chakrabarti and Chopra, 1973).

When reservoir boundaries are irregular, the entire dam-reservoir system is modelled by finite elements considering displacements as the unknowns for the dam and the pressures as the unknowns for the reservoir (Sharan, 1978; Zienkiewicz and Newton, 1969).

Obviously, finite element discretization cannot be applied if the reservoir is of infinite extent. In such a case, the energy loss in the outgoing waves has been modelled by assuming that beyond a certain length upstream of the dam the reservoir has a uniform rectangular section. Finite element discretization is then limited to the irregular portion of the reservoir. For the regular and infinite portion a continuum or a one dimensional finite element solution is obtained. Compatibility of pressures and pressure derivatives is then enforced at the interface of the regular and irregular sections of the reservoir to complete the solution (Hall and Chopra, 1982a and 1982b).

A flexible foundation affects the response in two ways. First, it dissipates energy by partially absorbing the hydrodynamic pressure waves impinging on the bottom of the reservoir, and second, it modifies the free field ground motion at the base of the dam. These effects can be considered either by modelling the foundation material as an elastic half space or by using finite elements to represent the flexible foundation (Chopra and Chakrabarti, 1981; Sharan, 1978). The fluid-reservoir interaction effect has also been accounted for in an approximate manner by using a simplified boundary condition at the interface which models the energy dissipated by partial absorption of the incident pressure waves (Hatamo and Nakagawa, 1972; Chopra and Chakrabarti, 1981).

Recently the boundary element method has been applied to the analysis of hydrodynamic forces in a two dimensional (Hanna and Humar, 1982) as well as a three dimensional finite reservoir (Humar, 1985; Humar and Jablonski, 1986). For infinite reservoirs, solutions have been obtained by omitting the far boundary. Although the procedure may give reasonable estimates of the hydrodynamic pressures, the results are sensitive to the length of reservoir boundary discretized and convergence is not easily achieved. In the present work a special boundary condition obtained by assuming that the reservoir is of regular section beyond a certain length upstream of the dam and obtaining a solution for the regular portion of the dam by classical method or a one dimensional finite element discretization (Hall and Chopra, 1982a), has been incorporated in the boundary element formulation. Also, a special boundary condition to account for foundation damping has been included. The BEM so modified is an efficient tool for analytical solution of the hydrodynamic pressures including the effect of radiation and foundation damping.

In the results presented here, the dam has been assumed to be rigid and undergoing a harmonic motion. The method can be easily extended to the computation of hydrodynamic pressure for a prescribed motion of the flexible dam face. Solution of the complete dam-reservoir system for an arbitrary ground motion would require the use of substructure approach along with an analysis in the frequency domain employing Fast Fourier Transform method.

2 FORMULATION OF BOUNDARY INTEGRAL EQUATION

For non-viscous but compressible water undergoing small amplitude two dimensional motion the following equations of motion apply, provided the effect of free waves at the surface is ignored:

$$\frac{\partial^2 u}{\partial t^2} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} \tag{1a}$$

$$\frac{\partial^2 v}{\partial t^2} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} \tag{1b}$$

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 \bar{p}}{\partial t^2} \tag{1c}$$

in which, x and y = the cartesian coordinates as shown in Fig. 1; u and v = respectively the x and y displacements of a particle of water; t = time variable; ρ = mass of water per unit volume; $\bar{p}$ = the hydrodynamic pressure ; and c = the velocity of sound in water.

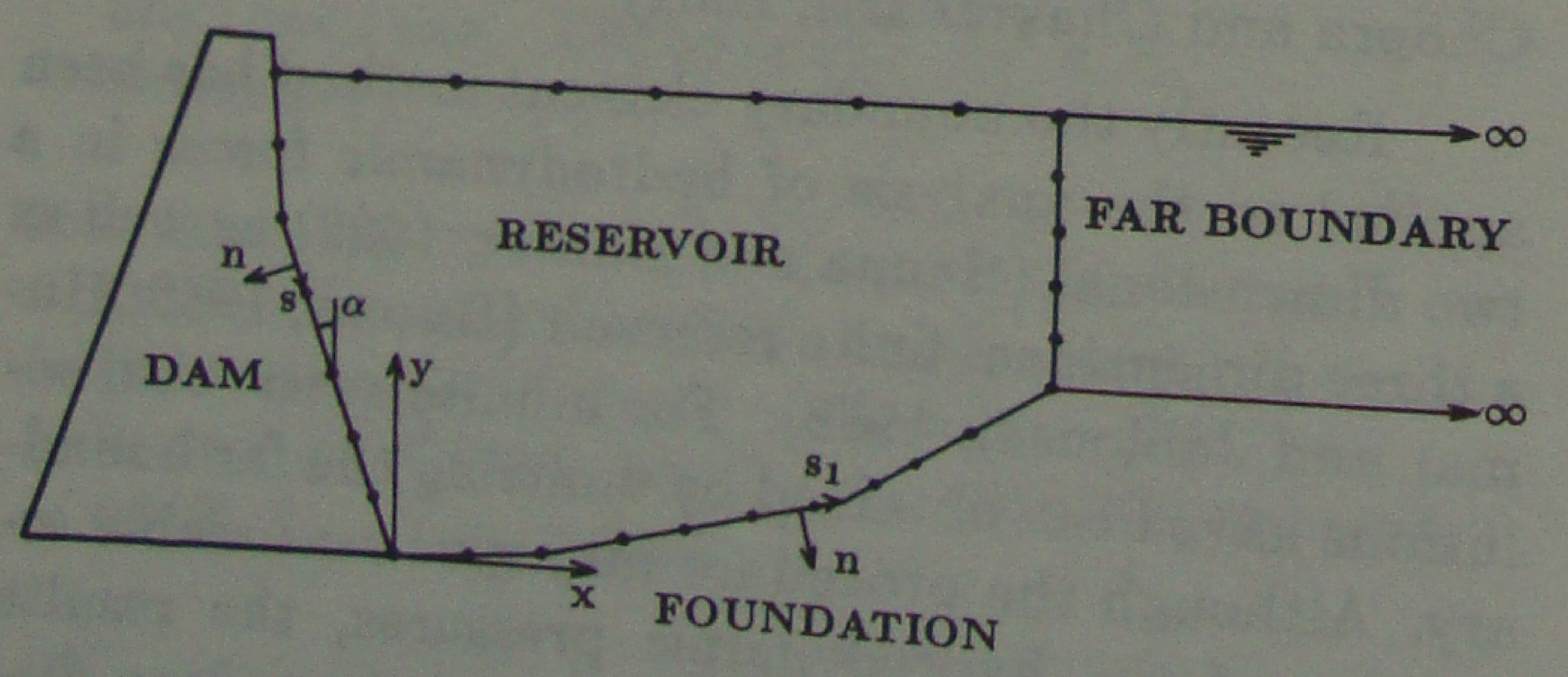


Fig. 1 Dam-reservoir-foundation system

For a harmonic motion of the dam, the hydrodynamic pressure can be represented by $\bar{p} = p(x, y, \omega)\, e^{i\omega t}$ where ω is the exciting frequency. Substitution into Eq. 1c then gives:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + k^2 p = 0 \tag{2}$$

where $k = \omega/c$ is the wave number.

For a numerical solution of the problem represented by Eq. 2, a weighted residual formulation is used in which the governing equation is satisfied in an average sense over the domain. This gives equation:

$$\int\!\!\int_A \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + k^2 p\right) p^* dx dy = 0 \tag{3}$$

in which p^* is the weighting function.

Partial integrations of Eq. 3 give:

$$\int\!\!\int_A \left(\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + k^2 p^*\right) p\, dx dy = \int_C p\, q^* dc - \int_C p^* q\, dc \tag{4}$$

in which $q^* = \partial p^*/\partial n$, $q = \partial p/\partial n$, C = boundary of the water reservoir, and n = the outward normal to the boundary.

The weighting function p^* is now chosen to satisfy the following equation:

$$\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + k^2 p^* = \Delta^i \tag{5}$$

in which Δ^i = a Dirac Delta function centered at point i having coordinates (x_i, y_i).

Substitution of Eq. 5 in Eq. 4 gives:

$$c_i\ p_i = \int_C p\, q^* dc - \int_C p^* q\, dc \tag{6}$$

in which $c_i = 1$ for a point inside the water reservoir, $c_i = 1/2$ for a point on the smooth boundary of the water reservoir, $c_i = 0$ for a point outside the water reservoir, and p_i = the hydrodynamic pressure at point i.

When i is located at a corner on the boundary, as shown in Fig. 2, the value of c_i can be shown to be given by:

$$c_i = 1 - \frac{\beta}{2\pi} \tag{7}$$

where β is the external angle at the corner.

Equation 6 provides a means of determining the unknown pressure at any point i within the domain in terms of the integrals of pressure and pressure derivative values on the boundary. While some of these boundary values are specified, others must be determined before Eq. 6 can be used. In boundary element formulation, Eq. 6 itself is used to determine the unknowns on the boundary. To achieve this, the boundary of the domain is divided into a series of segments and the parameters p and q are assumed to vary in a prescibed manner over a given segment. In constant boundary element formulation, the segments

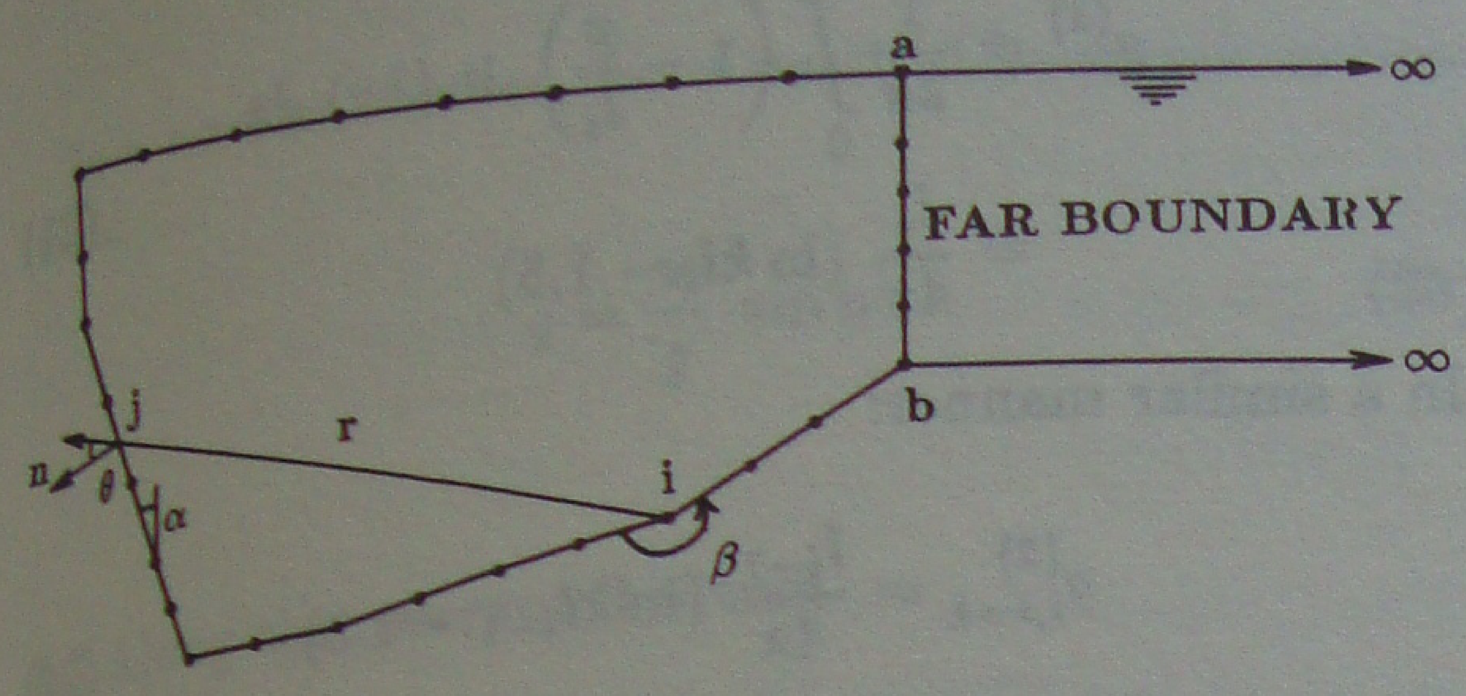


Fig. 2 Boundary element discretization of a reservoir

are straight lines, and p and q are assumed to be equal to their values at the midpoint of the segment. This midpoint is referred to as a node. In a linear element formulation, individual segments again consist of straight lines but parameters p and q are assumed to vary linearly over a segment and are functions of their values at the end points of the segment. The end points of the segment are referred to as the nodes so that each segment is bounded by two nodes. Equation 6 is now written for point i located at each node on the discretized boundary, and takes the form:

$$-\left(1-\frac{\beta}{2\pi}\right) p_i + \sum_{j=1}^{N}\int_{s_j} p\, q^*\, ds_j = \sum_{j=1}^{N}\int_{s_j} p^* q\, ds_j \quad (8)$$

in which s_j is the boundary of the jth element.

Using linear boundary elements and referring to Fig. 3 which shows the jth element, we have:

$$p = [\phi_1 \quad \phi_2]\begin{Bmatrix} p_j \\ p_{j+1}\end{Bmatrix}$$
$$q = [\phi_1 \quad \phi_2]\begin{Bmatrix} q_j \\ q_{j+1}\end{Bmatrix} \quad (9)$$

in which ϕ_1 and ϕ_2 are the shape functions given by:

$$\phi_1 = 1/2\,(1-\xi)$$
$$\phi_2 = 1/2\,(1+\xi) \quad (10)$$

and ξ is a coordinate along the boundary element.

Equation 8 can now be expressed as

$$-\left(1-\frac{\beta}{2\pi}\right) p_i + \sum_{j=1}^{N}\int_{s_j}[\phi_1 \quad \phi_2]\begin{Bmatrix} p_j \\ p_{j+1}\end{Bmatrix} q^* ds_j = \sum_{j=1}^{N}\int_{s_j}[\phi_1 \quad \phi_2]\begin{Bmatrix} q_j \\ q_{j+1}\end{Bmatrix} p^* ds_j \qquad i = 1,2,\cdots,N \quad (11)$$

Introducing the notations:

$$\int_{s_j}\phi_1\, q^* ds_j = h_{ij}^{(1)}$$
$$\int_{s_j}\phi_2\, q^* ds_j = h_{ij}^{(2)}$$
$$\int_{s_j}\phi_1\, p^* ds_j = g_{ij}^{(1)} \quad (12)$$
$$\int_{s_j}\phi_2\, p^* ds_j = g_{ij}^{(2)}$$

Eq. 11 can be written as:

$$-\left(1-\frac{\beta}{2\pi}\right) p_i + \sum_{j=1}^{N}(h_{ij}^{(1)}\, p_j + h_{ij}^{(2)}\, p_{j+1}) = \sum_{j=1}^{N}(g_{ij}^{(1)}\, q_j + g_{ij}^{(2)}\, q_{j+1}) \qquad i = 1,2,\cdots,N \quad (13)$$

Assembly of the N equations represented by Eq. 13 gives, in matrix form:

$$\mathbf{H\,p = G\,q} \quad (14)$$

in which:

$$H_{ij} = h_{ij}^{(1)} + h_{(i,j-1)}^{(2)} \qquad i \neq j$$
$$H_{ii} = -\left(1-\frac{\beta}{2\pi}\right) + h_{ii}^{(1)} + h_{(i,i-1)}^{(2)}$$
$$G_{ij} = g_{ij}^{(1)} + g_{(i,j-1)}^{(2)}$$

3 FUNDAMENTAL SOLUTION

The solution of Eq. 5 is commonly referred to as the fundamental solution and can be shown to be (Hanna, 1982 ; Greenberg, 1971) given by:

$$p^* = \frac{1}{4}\, Y_0(kr) \quad (15)$$

in which Y_0 = Bessel function of zero order and second kind; r = the radial distance variable given by $r = \sqrt{(x-x_i)^2 + (y-y_i)^2}$.

4 EVALUATION OF BOUNDARY INTEGRALS

Using the fundamental solution given by Eq. 15, the boundary integrals in Eq. 12 can be evaluated as follows:

$$h_{ij}^{(1)} = \int_{s_j} \phi_1 \, q^* \, ds$$
$$= \frac{kl}{16} \int_{-1}^{+1} (1-\xi)\, Y_0'(kr)\, \cos\theta \, d\xi \qquad (16)$$

in which $Y_0'(kr)$ = the derivative of $Y_0(kr)$ with respect to kr, and θ = the angle between the outward normal n, and the radius vector r, as shown in Fig. 2.

The integral in Eq. 16 is evaluated numerically using say a four-point Gauss quadrature formula giving:

$$h_{ij}^{(1)} = \frac{kl_j}{16} \sum_{m=1}^{4} (1-\xi_m) Y_0'(kr)_m (cos\theta)_m w_m \qquad (17)$$

in which w_m = the weighting function used in the Gauss quadrature.

In a similar manner:

$$h_{ij}^{(2)} = \frac{kl_j}{16} \sum_{m-1}^{4} (1+\xi_m) Y_0'(kr)_m (\cos\theta)_m w_m$$
$$g_{ij}^{(1)} = \frac{l_j}{16} \sum_{m-1}^{4} (1-\xi_m) Y_0(kr)_m w_m \qquad (18)$$
$$g_{ij}^{(2)} = \frac{l_j}{16} \sum_{m-1}^{4} (1+\xi_m) Y_0(kr)_m w_m$$

For the special case of H_{ii}, the terms $h_{ii}^{(1)}$ and $h_{i,i-1}^{(2)}$ will vanish because of the orthogonality of r and n so that $cos\theta = 0$. This gives:

$$H_{ii} = -1 + \frac{\beta}{2\pi} \qquad (19)$$

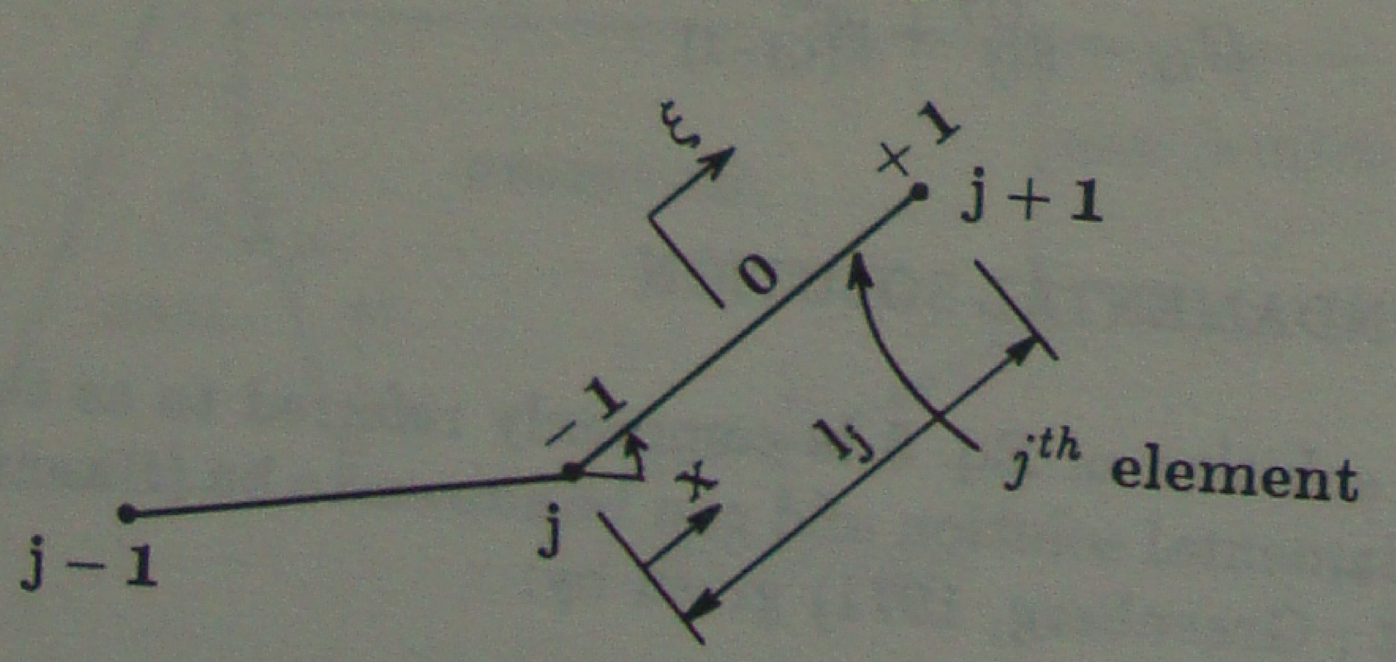


Fig. 3 Coordinate system used in the evaluation of boundary integrals

In the evaluation of G_{ij}, singular integral functions arise both when $j = i$ and $j = i+1$. Integrals containig singular terms are evaluated analytically using the approximation:

$$Y_0(kr) \simeq \frac{2}{\pi} \ln(kr) \quad \text{as} \quad r \to 0 \qquad (20)$$

Thus,

$$G_{ii} = g_{ii}^{(1)} + g_{i,i-1}^{(2)}$$

and referring to Fig. 3,

$$g_{ii}^{(1)} = \frac{1}{2\pi} \int_0^{l_i} \left(1 - \frac{x}{l_i}\right) \ln(kx)\, dx$$
$$= \frac{l_i}{4\pi} (\ln kl_i - 1.5) \qquad (21)$$

In a similar manner:

$$g_{i,i-1}^{(2)} = \frac{l_{i-1}}{4\pi} (\ln kl_{i-1} - 1.5) \qquad (22)$$

Also,

$$G_{i,i+1} = g_{i,i+1}^{(1)} + g_{ii}^{(2)} \qquad (23)$$

in which term $g_{ii}^{(2)}$ containing a singular integrand is evaluated as follows:

$$g_{ii}^{(2)} = \frac{1}{2\pi} \int_0^{l_i} x \, \ln(kx)\, dx$$
$$= \frac{l_i}{4\pi} (\ln kl_i - 0.5) \qquad (24)$$

5 BOUNDARY CONDITIONS

After the elements of matrices **G** and **H** have been obtained, the following boundary conditions are applied:

5.1 Along surface of the reservoir

The hydrodynamic pressure should vanish at the free surface of the reservoir i.e. $\bar{p}(x, H, t) = 0$ which gives $p(x, H) = 0$.

5.2 Along the dam face

$$\frac{\partial \bar{p}}{\partial n}(s,t) = -\frac{w}{g}\, a_n(s,t)\, e^{i\omega t} \qquad (25)$$

in which w is the unit weight of water, s is the coordinate along the dam face, and a_n is the component of harmonic exciting acceleration amplitude along the outward normal. When the excitation is a horizontal acceleration $e^{i\omega t}$,

$$a_n = -\cos\alpha \tag{26}$$

in which α is the angle the dam face makes with the vertical. Boundary condition given by Eq. 25 can thus be expressed as:

$$q = \frac{\partial p}{\partial n}(\omega, t) = \frac{w}{g}\cos\alpha \tag{27}$$

In a similar manner, for a vertical harmonic excitation:

$$q = \frac{w}{g}\sin\alpha \tag{28}$$

5.3 Along the reservoir bottom

In this case, the boundary condition is adapted to include, in an approximate manner, the effect of wave absorption. The derivation has been presented by Hall and Chopra (1982a) and will not be repeated here. The resulting condition is

$$q = \frac{\partial p}{\partial n}(s', \omega) = -\frac{w}{g}\, a_n(s') - i\omega\gamma\, p(s', \omega) \tag{29}$$

in which s' = the coordinate along the reservoir bottom, $\gamma = w/(w_r C_r)$ = the foundation damping coefficient, w_r = unit weight of the foundation material, C_r = compression wave velocity in the foundation material and $a_n(s')$ = the harmonic excitation amplitude along the outward normal. For a horizontal excitation

$$a_n(s') = \sin\phi \tag{30}$$

while for a vertical excitation

$$a_n(s') = -\cos\phi \tag{31}$$

in which ϕ is the angle that the reservoir bottom makes with the horizontal.

The second term on the right hand side of Eq. 29 represents the effect of foundation damping. For rigid foundation, $\gamma = 0$ and the damping is absent. Foundation damping can also be defined in terms of a wave reflection coefficient α_r which is related to γ as follows (Hall and Chopra 1982a):

$$\gamma = \frac{1}{C}\frac{1-\alpha_r}{1+\alpha_r} \tag{32}$$

5.4 Along the far boundary

For a finite reservoir, the condition at the far boundary depends on whether that boundary is fixed or free to move. For an infinite reservoir, the far boundary is located at the end of the irregular region of the reservoir. Beyond this boundary, the reservoir is assumed to have a regular rectangular section extending to infinity. The energy loss due to radiation damping in the infinite region is accounted for by imposing a special boundary condition at the far boundary. The derivation of the boundary condition closely parallels that presented by Hall and Chopra (1982a) and is briefly summarized in the following section.

6 RADIATION BOUNDARY CONDITION

The infinite section of the reservoir is shown in Fig. 4. When the excitation is only a horizontal ground motion, the hydrodynamic pressure in this section is governed by Eq. 2 with the following boundary conditions:

$$\begin{aligned} &\frac{\partial p}{\partial x}(0, y, \omega) = q_f \\ &\frac{\partial p}{\partial y}(x, 0, \omega) = i\omega\gamma\, p(x, 0, \omega) \\ &p(x, H, \omega) = 0 \end{aligned} \tag{33}$$

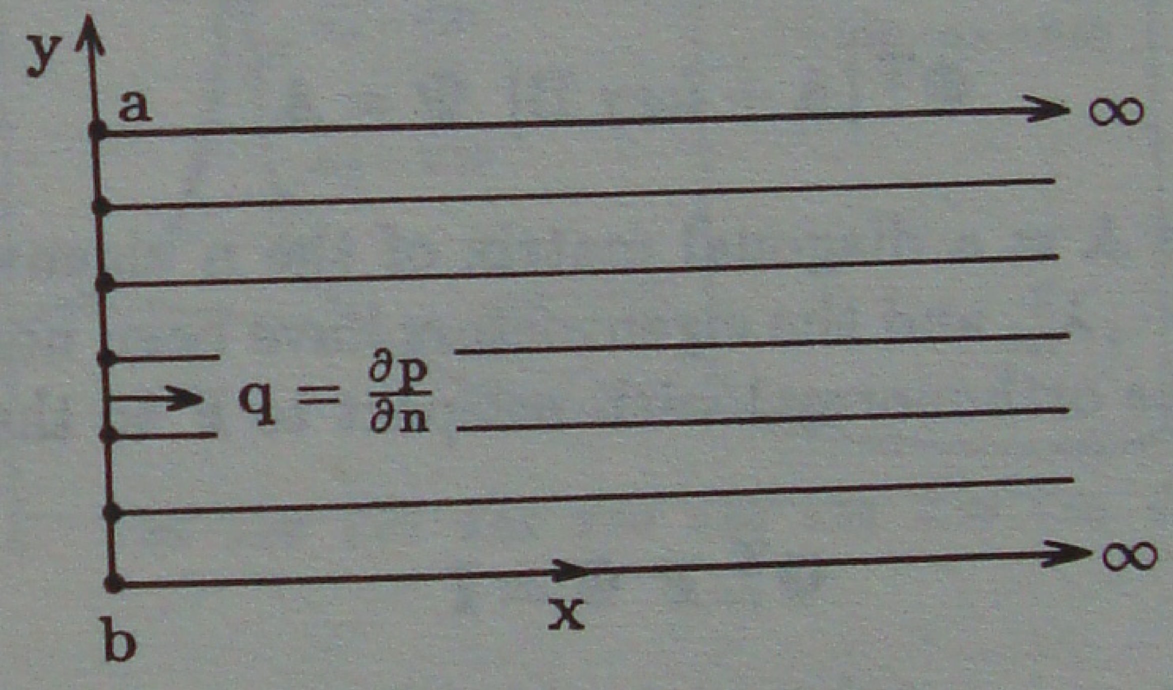


Fig. 4 The infinite but regular section of the reservoir

The governing equation can be solved by a seperation of variables

$$p = p_x p_y \tag{34}$$

in which p_x satisfies the equation

$$\frac{d^2 p_x}{dx^2} - \kappa^2 p_x = 0 \tag{35}$$

and p_y satisfies the equation

$$\frac{d^2 p_y}{dy^2} + \lambda^2 p_y = 0 \tag{36}$$

with the boundary condition

$$p_y = 0 \qquad \text{at } y = H$$
$$\frac{dp_y}{dy} = i\omega\gamma\, p_y \qquad \text{at } y = 0 \tag{37}$$

In Eqs. 34 and 35, κ = the seperation constant and λ is given by

$$\lambda^2 = k^2 + \kappa^2 \tag{38}$$

Equation 36 is solved by a one dimensional finite element discretization. The governing equation is thus expressed in discrete form as follows (Hall and Chopra 1982a).

$$[\mathbf{A} + i\omega\gamma\, \mathbf{B}]\ \mathbf{p_y} = \lambda^2\ \mathbf{F}\ \mathbf{p_y} \tag{39}$$

in which $\mathbf{p_y}$ = the vector of pressure values at the nodes. The surface node is excluded from the formulation because the pressure value is known to be zero there. Also, matrix $\mathbf{B}$ has only one non-zero value, that on the diagonal corresponding to the base node b.

Equation 39 represents an eigenvalue problem which for non-zero γ leads to complex eigenvalues and eigenvectors. If the matrix of n eigenvectors obtained from Eq. 38 is represented by $\mathbf{\Psi}$, the following relationships are satisfied:

$$\mathbf{\Psi^T}\,[\mathbf{A} + i\omega\gamma\, \mathbf{B}]\ \mathbf{\Psi} = \mathbf{\Lambda} \tag{40}$$

in which $\mathbf{\Lambda}$ = a diagonal matrix of the n eigenvalues $\lambda_1^2, \lambda_2^2, \cdots, \lambda_n^2$, and the eigenvectors have been normalized to be orthonormal with resepect to $\mathbf{F}$ so that

$$\mathbf{\Psi^T}\ \mathbf{F}\ \mathbf{\Psi} = \mathbf{I} \tag{41}$$

The solution to Eq. 35 can now be written as

$$p_x = e^{-\kappa_n x} \tag{42}$$

in which $\kappa_n = \sqrt{\lambda_n^2 - k^2}$.

When κ is real valued, it is taken with a positive sign to ensure that p_x tends to zero as $x \to \infty$. When κ is imaginary, both its real and its imaginary part should be positive. A positive real part will imply that the wave amplitude decays with increasing x. A positive imaginary part will account for the fact the waves move away towards infinity and no reflected waves are present.

The general solution for pressure vector $\mathbf{p}$ can now be expressed as

$$\mathbf{p} = \sum_{n=1}^{N} \eta_n\ \psi_n\ \ e^{-\kappa_n x} \tag{43a}$$

or

$$\mathbf{p} = \mathbf{\Psi}\ \mathbf{E}\ \eta \tag{43b}$$

in which η = a vector of nodal coordinates and $\mathbf{E}$ = is a diagonal matrix of terms $e^{-\kappa_n x}$. Taking the origin at the far boundary and differentiating Eq. 43

$$\mathbf{q} = -\mathbf{\Psi}\ \mathbf{K}\ \eta \tag{44}$$

in which $\mathbf{K}$ is a diagonal matrix with elements $\kappa_1, \kappa_2, \cdots, \kappa_N$.

When the excitation is a vertical ground motion, the hydrodynamic pressure is governed by Eq. 2 with the boundary conditions

$$\frac{\partial p}{\partial n}(0, y, \omega) = 0$$
$$\frac{\partial p}{\partial y}(x, 0, \omega) = -\frac{w}{g}\ a_y + i\omega\gamma\ p(x, 0, \omega) \tag{45}$$
$$p(x, H, \omega) = 0$$

The problem is one dimensional in y coordinate and x variation is absent so the governing equation becomes

$$\frac{d^2p}{dy^2} + k^2\ p = 0 \tag{46}$$

A numerical solution of governing equation with finite element discretization gives

$$\left[\mathbf{A} + i\omega\gamma\, \mathbf{B} - k^2\mathbf{F}\right] \mathbf{p} = \frac{w}{g}\ \mathbf{d} \tag{47}$$

in which $\mathbf{A}$, $\mathbf{B}$, $\mathbf{F}$ are the same matrices as defined in Eq. 39 and $\mathbf{d}$ is a vector with one non-zero term equal to a_y corresponding to the base node b. Equation 47 can be solved for $\mathbf{p}$ by using an eigenvector expansion in which the complex valued eigenvalues λ and eigenvectores $\mathbf{\Psi}$ determined from Eq. 38 are used. The vector of nodal pressures is then given by

$$\mathbf{p} = \frac{w}{g}\ \mathbf{\Psi}\ \mathbf{K}^{-2}\ \mathbf{\Psi}^T\ \mathbf{d} \tag{48}$$

Combining Eqs. 43b and 48, the total pressure at the far boundary is given by

$$\mathbf{p} = \mathbf{\Psi}\ \eta + \frac{w}{g}\ \mathbf{\Psi}\ \mathbf{K}^{-2}\ \mathbf{\Psi}^T\ \mathbf{d} \tag{49}$$

Now let the matrix Eq. 14 be expressed in a partitioned form as follows:

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} \\ \mathbf{H}_{31} & \mathbf{H}_{32} & \mathbf{H}_{33} \end{bmatrix} \begin{Bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{Bmatrix} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{Bmatrix} \tag{50}$$

in which p_2 and q_2 represent, respectively, the pressures and pressure derivatives at the nodes on the reservoir bottom while p_3 and q_3 are respectively the pressures and pressure derivatives at the nodes on the far boundary.

Substitution for q_2 from Eq. 29, p_3 from Eq. 49 and q_3 from Eq. 44 leads to

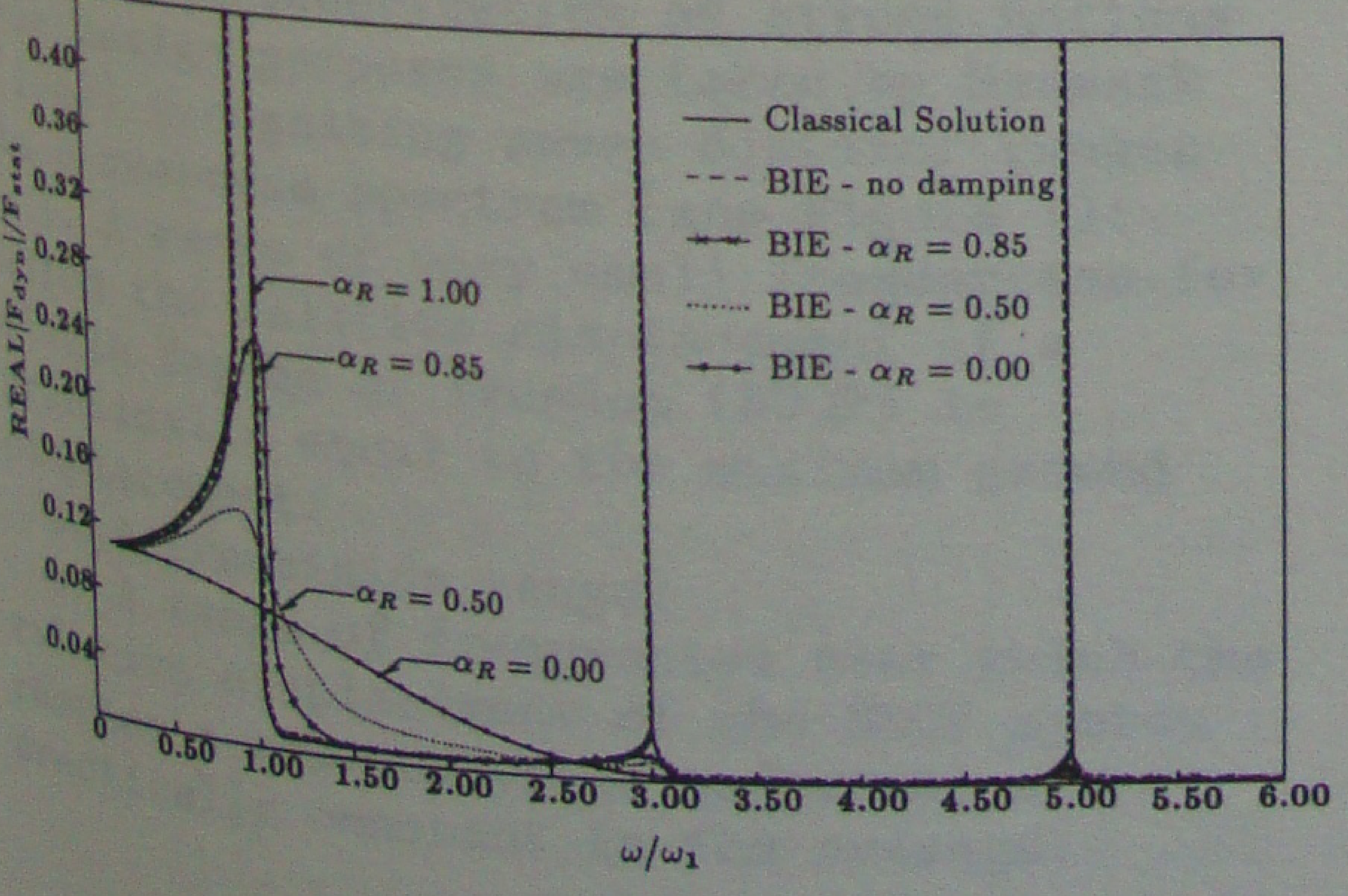
$$\begin{bmatrix} H_{11} & H_{12} + i\omega\gamma G_{12} & H_{13}\Psi + G_{13}\Psi K \\ H_{21} & H_{22} + i\omega\gamma G_{22} & H_{23}\Psi + G_{23}\Psi K \\ H_{31} & H_{32} + i\omega\gamma G_{32} & H_{33}\Psi + G_{33}\Psi K \end{bmatrix} \times$$

$$\begin{Bmatrix} p_1 \\ p_2 \\ \eta \end{Bmatrix} = \begin{Bmatrix} G_{11}q_1 - \frac{w}{g}G_{12}a_n - \frac{w}{g}H_{13}\Psi K^{-2}\Psi^T d \\ G_{21}q_1 - \frac{w}{g}G_{22}a_n - \frac{w}{g}H_{23}\Psi K^{-2}\Psi^T d \\ G_{31}q_1 - \frac{w}{g}G_{32}a_n - \frac{w}{g}H_{33}\Psi K^{-2}\Psi^T d \end{Bmatrix} \quad (51)$$

Equations 51 can be solved for all unknowns on the boundary including the nodal pressures on the dam face.

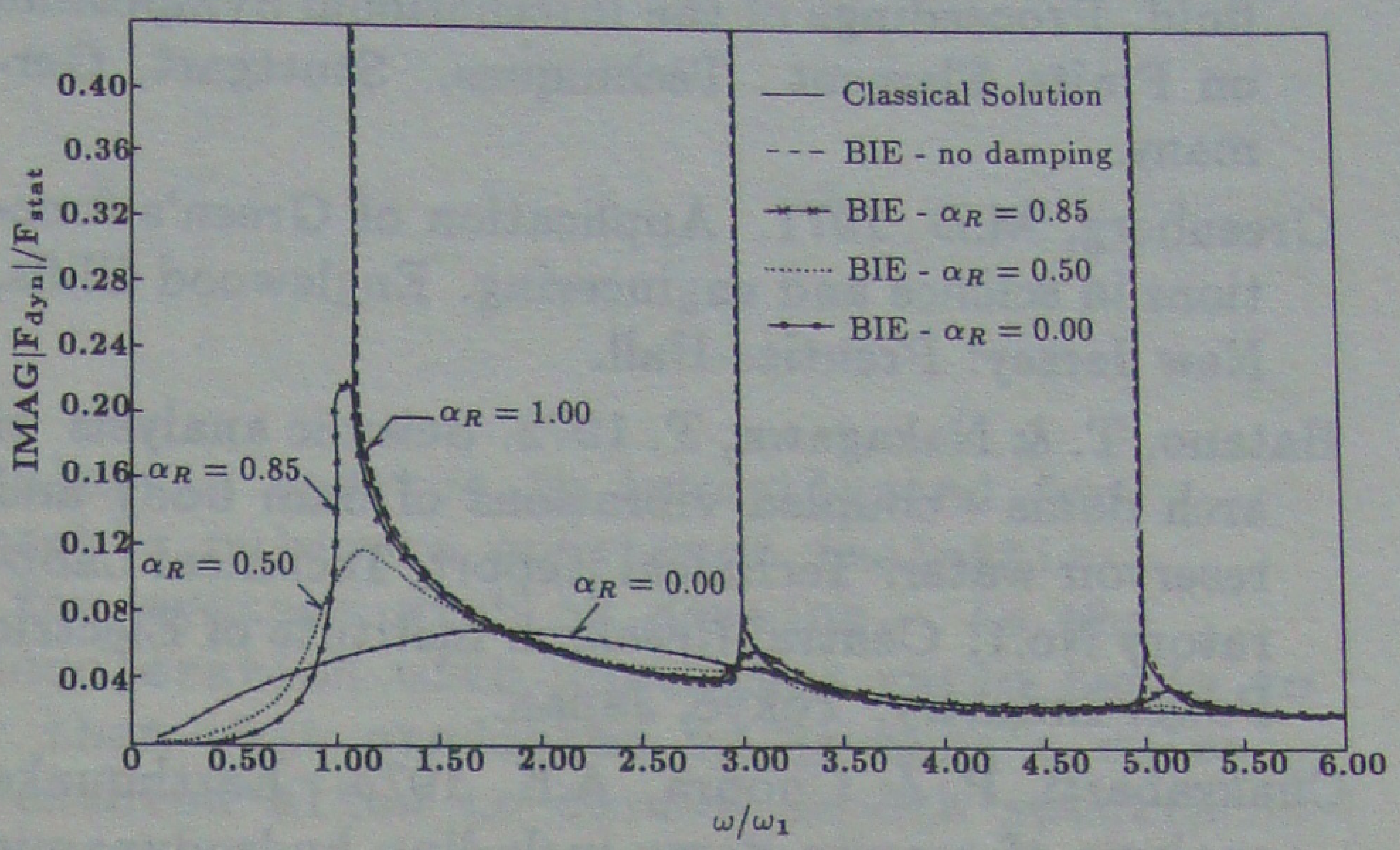
7 ANALYTICAL RESULTS

A limited amount of the results obtained from analytical studies based on the procedure described above are presented here. For the purpose of these studies the dam is assumed to be rigid, with a vertical face, the reservoir bottom is assumed to be horizontal, the reservoir extends to infinity and is excited by a horizontal harmonic motion of the dam. The resulting values of the hydrodynamic pressures on the dam are plotted in Fig. 5 for a range of values of the exciting frequency. The following data is used in the computation of the results shown: height of reservoir, $H = 100\ m$; velocity of sound in water, $c = 1440\ m/s$; the mass density of water; $\rho = 1\ tonne/m^3$. The far boundary of the reservoir was placed at a distance of 200 m from the dam face. Results are presented for both rigid as well as flexible foundation bottom, the damping provided by the base alluvium being represented by the reflection coefficient; a reflection coefficient $\alpha_r = 1$ representing a rigid case.

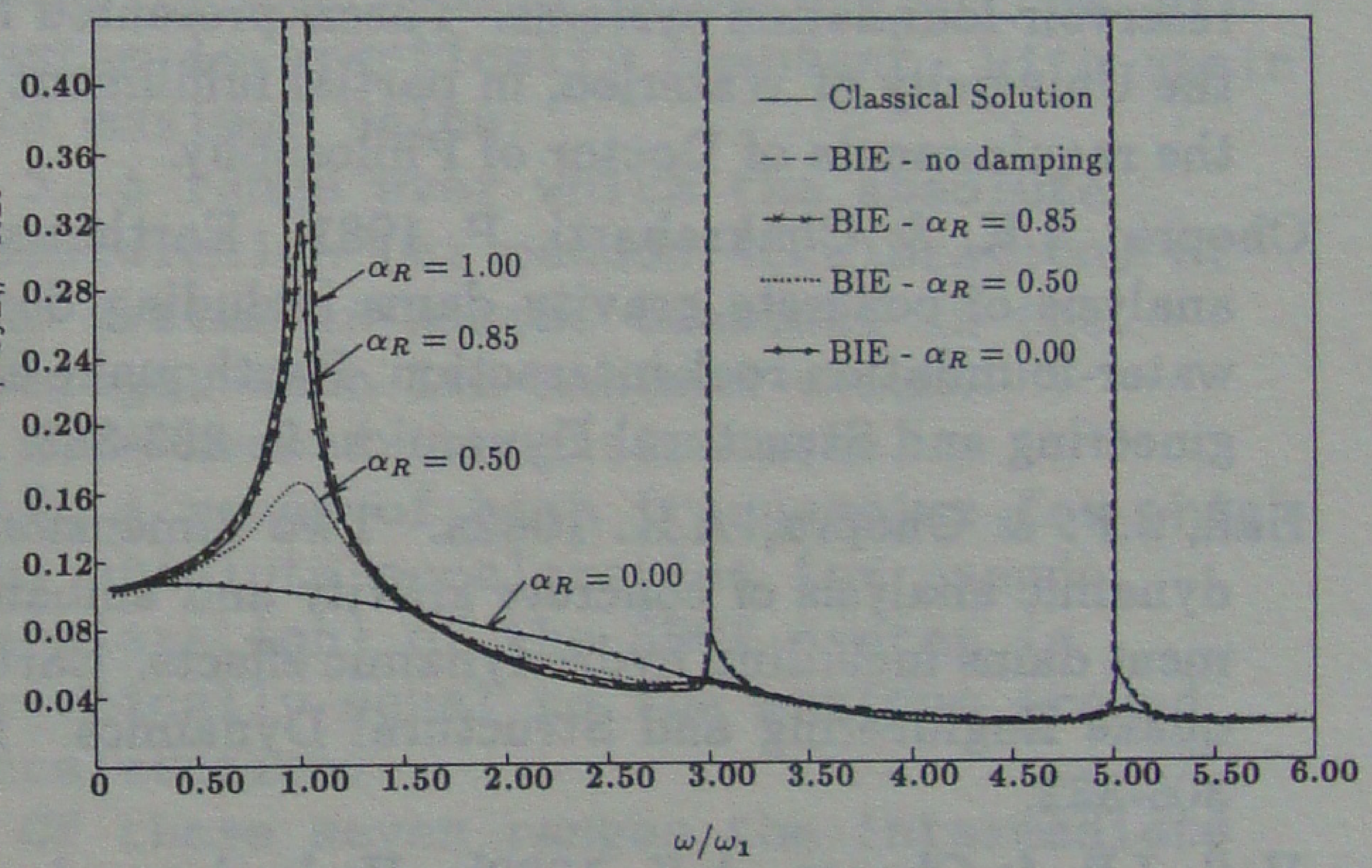


(a) Real part

Classical solutions exist for this simple geometry for a rigid base soil. The comparison between the classical and boundary element results is very good. Foundation damping significantly modifies the response near resonance. However, away from the resonant frequency the effect of foundation damping is small.



(b) Imaginary part



(c) Absolute value

Fig. 5 Hydrodynamic forces on a dam face due to harmonic excitation

8 SUMMARY AND CONCLUSIONS

The boundary element method is seen to be an effective procedure for evaluating the hydrodynamic pressures in a reservoir. If compared to a finite element method, it has the advantage of reducing the dimensionality of the problem by one. However, the matrices involved in the BEM are unsymmetric and fully populated. Also, the evaluation of Bessel functions involved in the fundamental solution for a 2D case takes substantial computation time. These observations would indicate that BEM would be very advantegeous for a 3D reservoir problem where a reduction in dimensionality from 3D to 2D and the simpler nature of the fundamental solution would make the BEM a very efficient procedure.

REFERENCES

Zienkiewicz, O.C. & Newton, R.Z. 1969. Coupled vibrations of structure submerged in a compressible fluid. Proceedings of the International Symposium on Finite Element Techniques. Stuttgart, Germany.

Greenberg, M.D. 1971. Application of Green's functions in science and engineering. Englewood Cliffs, New Jersey: Prentice-Hall.

Hatano, T. & Nakagawa, T. 1972. Seismic analysis of arch dams - coupled vibrations of dam body and reservoir water. Technical Report, Technical Laboratory No.1. Central Research Institute of Electric Power Industry. Tokyo, Japan.

Chakrabarti, P. & Chopra, A.K. 1973. Earthquake analysis of gravity dams including hydrodynamic interaction. Earthquake Engineering and Structural Dynamics. 2: 143-160.

Sharan, K.S. 1978. Earthquake response of dam-reservoir-foundation systems. Thesis presented for the University of Waterloo, in partial fulfillment of the requirements of Doctor of Philosophy.

Chopra, A.K. & Chakrabarti, P. 1981. Earthquake analysis of concrete gravity dams including dam-water-foundation rock interaction. Earthquake Engineering and Structural Dynamics. 9: 363-383.

Hall, J.F. & Chopra, A.K. 1982a. Two dimensional dynamic analysis of concrete gravity and embankment dams including hydrodynamic effects. Earthquake Engineering and Structural Dynamics. 10: 305-332.

Hall, J.F. & Chopra, A.K. 1982b. Hydrodynamic effects in the dynamic response of concrete gravity dams. Earthquake Engineering and Structural Dynamics. 10: 333-345.

Hanna, Y.G. & Humar, J.L. 1982. Boundary element analysis of fluid domain. Journal of the Engineering Mechanics Division, ASCE. No. EM2. 108:436-450.

Hanna, Y.G. 1982. Application of boundary element method to certain problems in structural dynamics. Thesis presented for Carleton University, in partial fulfillment of the requirements of Master in Engineering.

Humar, J.L. 1985. Boundary element analysis of hydrodynamic pressure on an arch dam. Proceedings of the Annual Conference of the Canadian Society of the Canadian Society for Civil Engineering and the 7th Canadian Hydrotechnical Conference. Saskatoon, Saskatchewan.

Humar, J.L. & Jablonski, A.M. 1986. Boundary element superposition analysis of reservoir motion. Proceedings of the 8th European Conference on Earthquake Engineering. Lisbon, Portugal.